PROPIEDADES DE LOS ÁNGULOS COMPUESTOS

Propiedades

I. Si:

$$K = A \cdot \text{Sen}x \pm B \cdot \text{Cos}x$$

$$\Rightarrow \begin{cases} K_{máx} = \sqrt{A^2 + B^2} \\ K_{mín.} = -\sqrt{A^2 + B^2} \end{cases}$$

Por ejemplo:

- $E = 3\text{Sen}x + 4\text{Cos}x$

$$\Rightarrow \begin{cases} E_{máx.} = \sqrt{3^2 + 4^2} = 5 \\ E_{mín} = -\sqrt{3^2 + 4^2} = -5 \end{cases}$$

- $Q = \sqrt{2}\text{Sen}x - \sqrt{7}\text{Cos}x$

$$\Rightarrow \begin{cases} Q_{máx.} = \sqrt{\sqrt{2}^2 + \sqrt{7}^2} = 3 \\ Q_{mín.} = -\sqrt{\sqrt{2}^2 + \sqrt{7}^2} = -3 \end{cases}$$

II. $\text{Tan}\alpha + \text{Tan}\beta + \text{Tan}\alpha \cdot \text{Tan}\beta \cdot \text{Tan}(\alpha + \beta) = \text{Tan}(\alpha + \beta)$

- $\text{Tan}12^\circ + \text{Tan}14^\circ + \text{Tan}12^\circ \cdot \text{Tan}14^\circ \cdot \text{Tan}26^\circ = \text{Tan}26^\circ$
  $12^\circ + 14^\circ$

- $\text{Tan}2x + \text{Tan}3x + \text{Tan}2x \cdot \text{Tan}3x \cdot \text{Tan}5x = \text{Tan}5x$
  $2x + 3x$

III.

Si: $\alpha + \beta + \theta = 180^\circ \vee 180^\circ n; (n \in \mathbb{Z})$

$\Rightarrow$
1) $\text{Tan}\alpha + \text{Tan}\beta + \text{Tan}\theta = \text{Tan}\alpha \cdot \text{Tan}\beta \cdot \text{Tan}\theta$
2) $\text{Cot}\alpha \cdot \text{Cot}\beta + \text{Cot}\alpha \cdot \text{Cot}\theta + \text{Cot}\beta \cdot \text{Cot}\theta = 1$

- $\text{Tan}40^\circ + \text{Tan}80^\circ + \text{Tan}60^\circ = \text{Tan}40^\circ \cdot \text{Tan}80^\circ \cdot \text{Tan}60^\circ$
  $(40^\circ + 80^\circ + 60^\circ = 180^\circ)$

- $\text{Cot}20^\circ \cdot \text{Cot}60^\circ + \text{Cot}60^\circ \cdot \text{Cot}100^\circ + \text{Cot}100^\circ \cdot \text{Cot}20^\circ = 1$
  $(20^\circ + 60^\circ + 100^\circ = 180^\circ)$

IV.

Si $x + y + z = 90^\circ \vee (2n+1)\frac{\pi}{2}$; $(n \in \mathbb{Z})$

1) $\text{Cot}x + \text{Cot}y + \text{Cot}z = \text{Cot}x \cdot \text{Cot}y \cdot \text{Cot}z$
2) $\text{Tan}x \cdot \text{Tan}y + \text{Tan}x \cdot \text{Tan}z + \text{Tan}y \cdot \text{Tan}z = 1$

- $\text{Cot}20^\circ + \text{Cot}60^\circ + \text{Cot}10^\circ = \text{Cot}20^\circ \cdot \text{Cot}60^\circ \cdot \text{Cot}10^\circ$
  $(20^\circ + 60^\circ + 10^\circ = 90^\circ)$

- $\text{Tan}20^\circ \cdot \text{Tan}42^\circ + \text{Tan}42^\circ \cdot \text{Tan}28^\circ + \text{Tan}28^\circ \cdot \text{Tan}20^\circ = 1$
  $(20^\circ + 42^\circ + 28^\circ = 90^\circ)$

Trabajando en clase

Integral

1. Calcula el mínimo valor de:
$P = 5\text{Sen}x - 12\text{Cos}x + 7$

2. Encuentra el valor de «x» si:
$\text{Tan}x + \text{Tan}2x + \text{Tan}x \cdot \text{Tan}2x \cdot \text{Tan}\,3x = \sqrt{3}$

3. En un triángulo ABC, simplifica:

$$Q = \frac{\text{Tan}A + \text{Tan}B + \text{Tan}C}{\text{Tan}A \cdot \text{Tan}B}$$

PUCP

4. Reduce:
$Q = 3 + \text{Tan}23^\circ + \text{Tan}22^\circ + \text{Tan}23^\circ \cdot \text{Tan}22^\circ$

Resolución:
$Q = 3 + \text{Tan}23^\circ + \text{Tan}22^\circ + \text{Tan}23^\circ \cdot \text{Tan}22^\circ \cdot (1)$
$(23^\circ + 22^\circ)$

$Q = 3 + \underbrace{\text{Tan}23^\circ + \text{Tan}22^\circ + \text{Tan}23^\circ \cdot \text{Tan}22^\circ \cdot \text{Tan}45^\circ}$
$Q = 3 + \text{Tan}45^\circ$
$Q = 3 + 1$
$Q = 4$

5. Calcula:

$$N = \frac{\text{Tan}32^\circ + \text{Tan}28^\circ + \sqrt{3}\text{Tan}32^\circ \cdot \text{Tan}28^\circ}{\text{Cot}60^\circ}$$

6. En un triángulo ABC $\text{Tan}A = 1$ y $\text{Tan}B = 2$. Calcula $\text{Tan}C$.

7. En un triángulo ABC $\text{Tan}A + \text{Tan}C = 3\text{Tan}B$
Obtén el valor de $E = \text{Tan}A \cdot \text{Tan}C$

UNMSM

8. En un triángulo ABC, simplifica:

$$R = \frac{\text{Cot}A + 2\text{Tan}B}{\text{Tan}B} + \frac{\text{Cot}B + 3\text{Tan}C}{\text{Tan}C} + \frac{\text{Cot}C + \text{Tan}A}{\text{Tan}A}$$

Resolución:
$$R = \frac{\text{Cot}A}{\text{Tan}B} + \frac{2\text{Tan}B}{\text{Tan}B} + \frac{\text{Cot}B}{\text{Tan}C} + \frac{3\text{Tan}C}{\text{Tan}C} + \frac{\text{Cot}C}{\text{Tan}A} + \frac{\text{Tan}A}{\text{Tan}A}$$

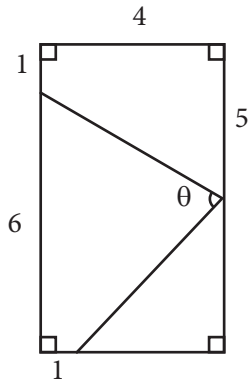
$R = \text{Cot}A \cdot \text{Cot}B + 2 + \text{Cot}B \cdot \text{Cot}C + 3 + \text{Cot}C \cdot \text{Cot}A + 1$
$R = \underbrace{(\text{Cot}A \cdot \text{Cot}B + \text{Cot}B \cdot \text{Cot}C + \text{Cot}C \cdot \text{Cot}A)} + 6$
$R = 1 + 6$
$R = 7$

9. Si $\alpha + \theta + \beta = 180^\circ$, calcula:

$$E = \frac{\text{Cot}\alpha + 6\text{Tan}\theta}{\text{Tan}\theta} + \frac{\text{Cot}\theta + 5\text{Tan}\beta}{\text{Tan}\beta} + \frac{\text{Cot}\beta + 4\text{Tan}\alpha}{\text{Tan}\alpha}$$

10. Calcula:
$N = 4 + \text{Tan}^2 10^\circ + 2\text{Tan}70^\circ \cdot \text{Tan}10^\circ$

11. Calcula Cotθ.



UNI

12. Señala el mayor valor de:

$Q = 2\sqrt{2}\,\text{Sen}(45^\circ + x) + \text{Cos}x$

Resolución:

$Q = 2\sqrt{2}(\text{Sen}45^\circ\text{Cos}x + \text{Cos}45^\circ\text{Sen}x) + \text{Cos}x$

$$Q = 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\text{Cos}x + \frac{1}{\sqrt{2}}\text{Sen}x\right) + \text{Cos}x$$

$Q = (2\text{Cos}x + 2\text{Sen}x) + \text{Cos}x$

$Q = 2\text{Sen}x + 3\text{Cos}x$

$Q_{\text{máx.}} = \sqrt{2^2 + 3^2}$

$Q_{\text{máx.}} = \sqrt{13}$

13. Calcula el mínimo valor de:

$Q = 5\text{Cos}(37^\circ + x) + \text{Sen}x$

14. Si $\text{Sen}(z + w) = \dfrac{6}{\sqrt{85}} \wedge \text{Cos}(x + y) = \dfrac{5}{\sqrt{34}}$

Determina:

$$E = \frac{5\text{Tan}x + 5\text{Tan}y + 3\text{Tan}x \cdot \text{Tan}y}{7\text{Tan}w + 7\text{Tan}z + 6\text{Tan}w \cdot \text{Tan}z}$$

Advertencia pre

$$\frac{\text{Sen}(x + y)}{\text{Sen}x \cdot \text{Sen}y} = \text{Cot}x + \text{Cot}y$$

$$\frac{\text{Sen}(x - y)}{\text{Sen}x \cdot \text{Sen}y} = \text{Cot}y - \text{Cot}x$$

$$\frac{\text{Sen}(x + y)}{\text{Cos}x \cdot \text{Cos}y} = \text{Tan}x + \text{Tan}y$$

$$\frac{\text{Sen}(x - y)}{\text{Cos}x \cdot \text{Cos}y} = \text{Tan}x - \text{Tan}y$$