



# OTROS PRODUCTOS NOTABLES

### 1. DESARROLLO DE UN BINOMIO AL CUBO

$$(a+b)^3 \equiv (a)^3 + 3(a)^2(b) + 3(a)(b)^2 + (b)^3$$

$$(a+b)^3 \equiv a^3 + b^3 + 3ab(a+b)$$

$$(a-b)^3 \equiv (a)^3 - 3(a)^2(b) + 3(a)(b)^2 - (b)^3$$

$$(a-b)^3 \equiv a^3 - b^3 - 3ab(a-b)$$

Ejemplos:

$$\begin{aligned} \diamond (x+2)^3 &= (x)^3 + 3(x)^2(2) + 3(x)(2)^2 + (2)^3 \\ &= x^3 + 6x^2 + 12x + 8 \end{aligned}$$

$$\begin{aligned} \diamond (x-3)^3 &= (x)^3 - 3(x)^2(3) + 3(x)(3)^2 - (3)^3 \\ &= x^3 - 9x^2 + 27x - 27 \end{aligned}$$

$$\diamond \text{ Si } x+y=3 \wedge xy=4, \text{ hallar: } x^3+y^3$$

**Resolución:**

Partimos de:

$$\begin{aligned} \underbrace{(x+y)}_3^3 &= x^3 + y^3 + 3 \underbrace{xy}_4 \underbrace{(x+y)}_3 \\ 3^3 &= x^3 + y^3 + 3(4)(3) \\ \therefore x^3 + y^3 &= -9 \end{aligned}$$

### 2. SUMA Y DIFERENCIA DE CUBOS

$$(a+b)(a^2-ab+b^2) = a^3+b^3$$

$$(a-b)(a^2+ab+b^2) = a^3-b^3$$

Ejemplos:

$$\diamond (x+3)(x^2-3x+9) = (x)^3 + (3)^3 = x^3 + 27$$

$$\begin{aligned} \diamond (2m-3n)(4m^2+6mn+9n^2) &= (2m)^3 - (3n)^3 \\ &= 8m^3 - 27n^3 \end{aligned}$$

### 3. DESARROLLO DE TRINOMIO AL CUADRADO Y AL CUBO

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ac)$$

$$(a+b+c)^3 = a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a)$$

**Ejemplo:**

$$\begin{aligned} \diamond (2x+3y+z)^2 &= (2x)^2 + (3y)^2 + (z)^2 + 2[(2x)(3y) + (2x)(z) + (3y)(z)] \\ (2x+3y+z)^2 &= 4x^2 + 9y^2 + z^2 + 2(6xy + 2xz + 3yz) \end{aligned}$$

### 4. IDENTIDADES CONDICIONALES

Si  $a+b+c=0$  se verifican:

$$\diamond a^2 + b^2 + c^2 = -2(ab+bc+ac)$$

$$\diamond (ab+bc+ac)^2 = (ab)^2 + (bc)^2 + (ac)^2$$

$$\diamond a^3 + b^3 + c^3 = 3abc$$

Ejemplo:

$$\text{Si } x+y+z=0; \text{ calcula: } E = \frac{x^3+y^3+z^3}{4xyz}$$

**Resolución:**

$$x^3 + y^3 + z^3 = 3xyz$$

$$\Rightarrow E = \frac{\cancel{3xyz}}{4xyz}$$

$$\therefore E = \frac{3}{4}$$

## TRABAJANDO EN CLASE

1. Desarrolla:

$$\diamond (a+2b)^3$$

$$\diamond (x-3y)^3$$

$$\diamond (x+2y)(x^2-2xy+4y^2)$$

$$\diamond (2m-n)(4m^2+2mn+n^2)$$

2. Si  $m+n=4$   $\wedge$   $mn=2$ .

Calcula el valor numérico de:  
 $m^3+n^3$

3. Si  $x-y=4$   $\wedge$   $x^3-y^3=-12$ .

Calcula el valor numérico de "xy".

4. Reduce:

$$A=(3x+2)(9x^2-6x+4)-(3x-2)(9x^2+6x+4)$$

**Resolución:**

De:

$$(a+b)(a^2-ab+b^2)=(a)^3+(b)^3$$

$$(a-b)(a^2+ab+b^2)=(a)^3-(b)^3$$

$$A=27x^3+8-(27x^3-8)$$

$$A=\cancel{27x^3}+8-\cancel{27x^3}+8$$

$$A=16$$

5. Calcula:

$$B=(\sqrt[3]{7}+\sqrt[3]{2})(\sqrt[3]{49}-\sqrt[3]{14}+\sqrt[3]{4})$$

$$+(\sqrt[3]{5}-\sqrt[3]{3})(\sqrt[3]{25}+\sqrt[3]{15}+\sqrt[3]{9})$$

6. Reduce:

$$A=(m+2)(m-2)(m^2-2m+4)(m^2+2m+4)+64$$

7. Si  $x+y=\sqrt[3]{2}$ ;  $xy=\sqrt[3]{4}$ .

Halla:

$$K=(x+y)^2(x^2-xy+y^2)^2-4x^3y^3$$

8. Si  $x+y+z=0$ , calcula el valor

$$\text{de: } M=\frac{x^3+y^3+z^3}{9xyz}$$

**Resolución:**

Por dato:  $x+y+z=0$  se cumple:  $x^3+y^3+z^3=3xyz$

$$\text{en el problema: } M=\frac{\cancel{3xyz}}{\cancel{9xyz}}=\frac{1}{3}$$

$$\therefore M=\frac{1}{3}$$

9. Si  $m+n+p=0$   $\wedge$   $mnp=5$ .

Calcula  $m^3+n^3+p^3$

10. Si  $\sqrt[3]{x}+\sqrt[3]{y}+\sqrt[3]{z}=0$

$$xyz=4$$

Calcula el valor de:

$$T=\left(\frac{x+y+z}{3}\right)^3$$

11. Si  $a+b+c=11$ , calcula el valor de:

$$A=\frac{(a-3)^3+(b-6)^3+(c-2)^3}{(a-3)(c-2)(b-6)}$$

12. Si  $a+b+c=10$ ;

$$a^2+b^2+c^2=60$$

Calcula:  $ab+bc+ac$

**Resolución**

Partimos de:

$$\underbrace{(a+b+c)^2}_{10}=\underbrace{a^2+b^2+c^2}_{60}+2(ab+bc+ac)$$

$$10^2=60+2(ab+bc+ac)$$

$$40=2(ab+bc+ac)$$

$$\therefore ab+bc+ac=20$$

13. Si  $a+b+c=8$

$$ab+bc+ac=15$$

Calcula  $a^2+b^2+c^2$

14. Si  $x+y+z=0$ , calcula:

$$M=\frac{x^3+y^3+z^3}{xyz}+\frac{x^2+y^2+z^2}{xy+xz+yz}$$