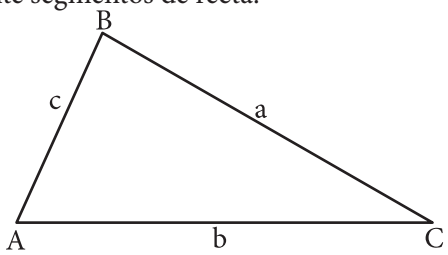
LOS TRIÁNGULOS

TRIÁNGULO RECTILÍNEO

Es aquel que se forma al unir tres puntos no colineales mediante segmentos de recta.



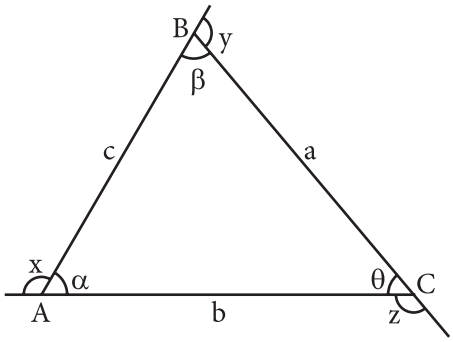
Elementos
Vértices: A, B y C
Lados: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

Notación
ΔABC: Se lee, triángulo ABC o de vértices A, B y C.

Cálculo del perímetro
$2p_{\Delta ABC} = a + b + c$

Si se sabe: AB = c, BC = a y CA = b

Propiedades fundamentales



a) $\boxed{\alpha + \beta + \theta = 180°}$

b) $\boxed{x + y + z = 360°}$

c) $\boxed{\alpha + \beta = z}$

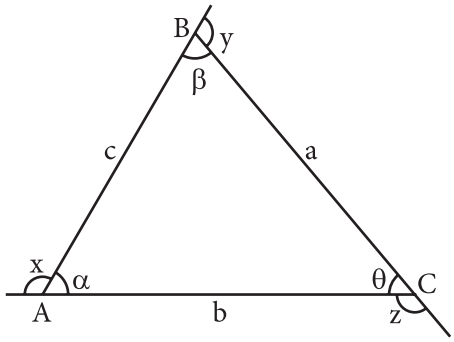
$\boxed{\beta + \theta = x}$

$\boxed{\alpha + \theta = y}$

d) Relación de correspondencia

$\boxed{\beta \geq \alpha \geq \theta}$

Si: $b \geq a \geq c$



e) Relación de existencia:

$\boxed{b - c < a < b + c}$

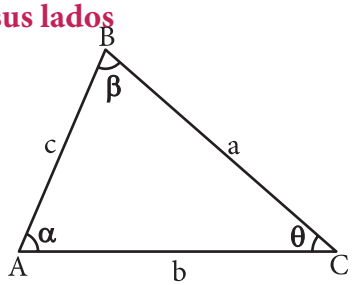
$\boxed{a - c < b < a + c}$

$\boxed{b - a < c < b + a}$

CLASIFICACIÓN DE LOS TRIÁNGULOS

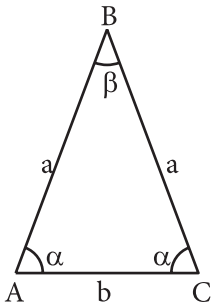
A. Según las medidas de sus lados

1. Δ Escaleno



$a \neq b \rightarrow \alpha \neq \beta$
$b \neq c \rightarrow \beta \neq \theta$
$c \neq a \rightarrow \theta \neq \alpha$

2. Δ Isósceles

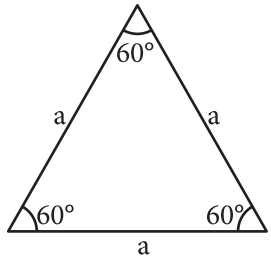


$a \neq b$

$\alpha \neq \beta$

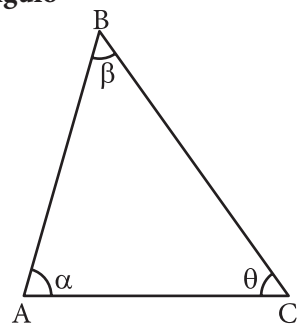
$\overline{AC}$: base

3. Δ Equilátero



B. Según las medidas de sus ángulos

1. Δ Acutángulo

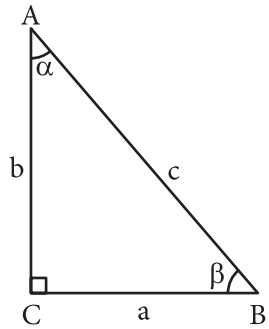


$\alpha < 90°$

$\beta < 90°$

$\theta < 90°$

2. Δ Rectángulo

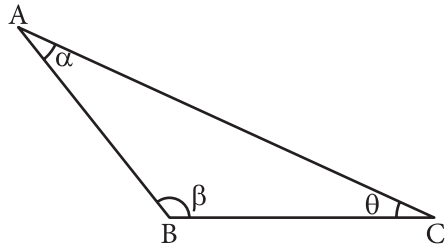


$\alpha + \beta = 90°$

$a^2 + b^2 = c^2$

(T. Pitágoras)

3. Δ Obtusángulo



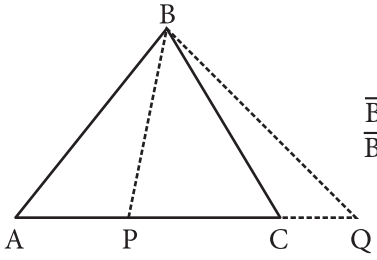
$\beta > 90°$

$\alpha < 90°$

$\theta < 90°$

LÍNEAS NOTABLES ASOCIADAS A LOS TRIÁNGULOS

Ceviana



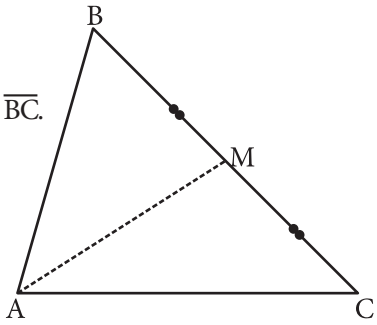
$\overline{BP}$: Ceviana interior.

$\overline{BQ}$: Ceviana exterior.

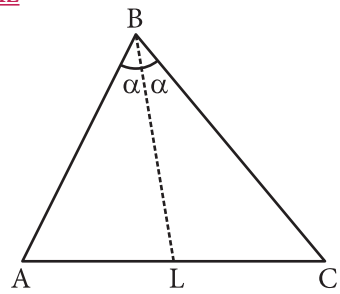
a) Mediana

Si: M es punto medio de $\overline{BC}$.

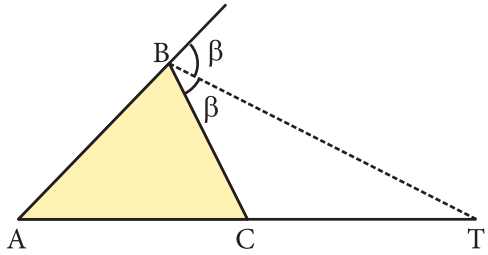
$\overline{AM}$: es mediana.



b) Bisectriz

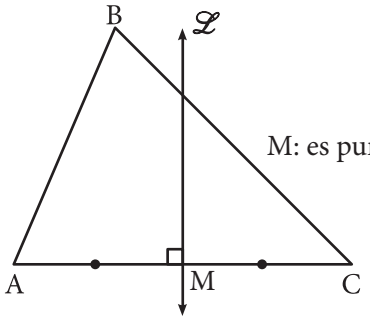


$\overline{BL}$: bisectriz interior



$\overline{BT}$: bisectriz exterior

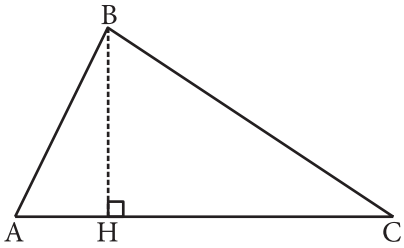
c) Mediatriz



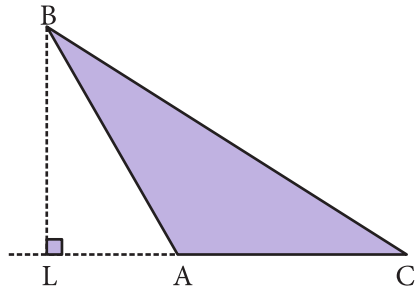
M: es punto medio.

$\overleftrightarrow{L}$: mediatriz de $\overline{AC}$

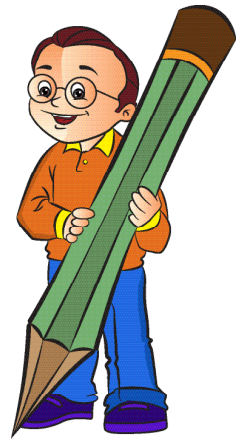
d) Altura



$\overline{BH}$: altura



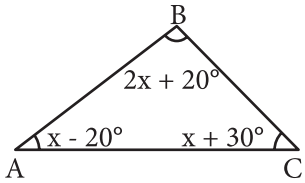
$\overline{BL}$: altura



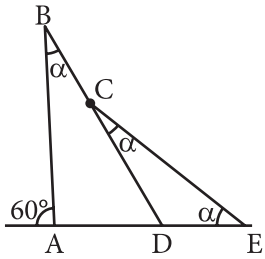
Trabajando en clase

Integral

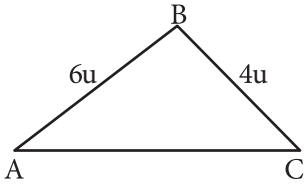
1. Calcula "x".



2. Calcula "α".

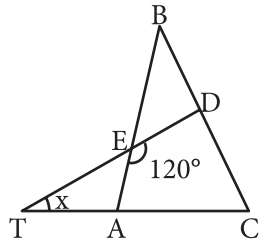


3. Si el triángulo ABC es escaleno, ¿cuántos triángulos se podrán formar con los valores enteros para las medidas del $\overline{AC}$?



PUCP

4. Calcula "x", si AB = BC y TC = TD



Solución:

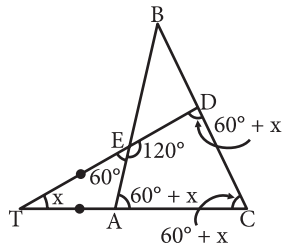
Como se forman isósceles, completamos de forma ordena.

$\Rightarrow m\angle A = m\angle C = 60° + x$

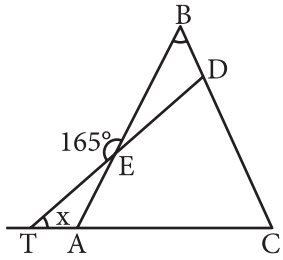
$m\angle C = m\angle D = 60° + x$

$\Rightarrow x + 60° + x + 60° + x = 180$.... ΔTDC

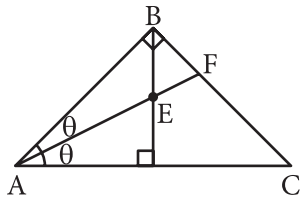
$x = 20°$



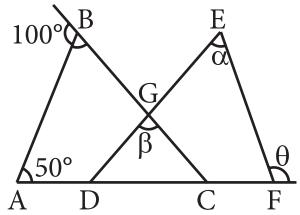
5. Calcula "x", si AB = BC y TC = TD.



6. Calcula FC si BC = 9m y BE = 4m.

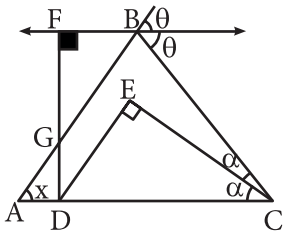


7. Calcula "α", si $\beta + \theta = 200°$.



UNMSM

8. Calcular "x"

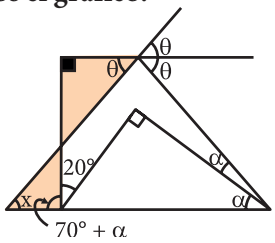


Solución:

Por ángulo exterior $2\theta = x + 2\alpha$

$\Rightarrow x = 2(\theta - \alpha)$

Acomodamos el gráfico:



Por propiedad:

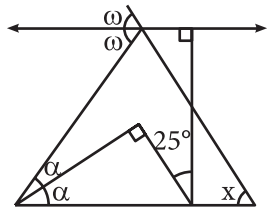
$x + 70° + \alpha = 90° + \theta$

$x + 70° = 90° + (\theta - \alpha)$

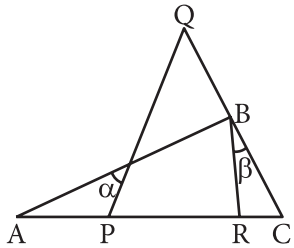
$(\times 2)\ 2x + 140° = 180° + 2\underbrace{(\theta - \alpha)}_{x}$

$x = 40°$

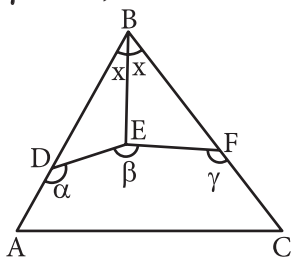
9. Calcula "x"



10. Si AB = AR y PQ = PC, calcula $k = \dfrac{2\alpha + 3\beta}{3\alpha - 3\beta}$



11. Si $\alpha + \beta + \gamma = 400°$, calcula "x".

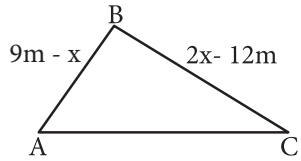


UNI

12. En un triángulo ABC; AB = 9m – x; BC = 2x – 12 m, además, m∠ A > m∠ C, calcula "x" si se sabe que es un número entero.

Solución:

El dibujo sería



Sabemos que a mayor ángulo se le opone mayor lado y viceversa.

$\Rightarrow 2x - 12 > 9 - x \rightarrow x > 7 \ \therefore x = 8m$

13. En un triángulo ABC, AB = 8m –x, BC = 3x-8 m; además m∠ A > m∠ C, calcula la suma de valores de "x" si es un número entero.

14. En un triángulo ABC, se cumple: AB = 2 m y AC = 32 m, calcula el perímetro del triángulo (en metros), sabiendo que es un número entero y que el ángulo "A" es obtuso.